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PRESSURE DROP AND VAPOR VOLUME WITH SUBCOOLED NUCLEATE BOILING

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Abstract—Reynolds' analogy between the transfer of heat and momentum is applied to forced-convection, subcooled boiling to predict the pressure drop. Experimental measurements of static pressure, vapor volume per unit area and heater wall temperature in an annular geometry permit the calculation of the local friction factor and the local Stanton number, from which it is demonstrated that the analogy gives a good approximation to the friction factor if a correction is made for the acceleration effects. However, this agreement is not found near the start of the heated length; local deviations of the measured quantities from those downstream are found in the entrance region.

NOMENCLATURE

A, area;
a, vapor volume per unit heater area;
C, circumferential length of heat-transfer surface;
c, specific heat;
D_h, hydraulic diameter of flow channel;
f, friction factor, $\tau_w/(\rho V^2/2g_c)$;
f_{app}, apparent friction factor, $(2g_c A_0/\rho V^2 C)(-\mathrm{d}p^*/\mathrm{d}x)$;
g, local acceleration of gravity;
g_c, dimensional constant, e.g. 32·17 ft lb/lbf s^2;
h, convective conductance, $q''/(T_w - T_b)$;
k, thermal conductivity;
N_{Pr}, Prandtl number, $\mu c/k$;
N_{Re}, Reynolds number, $D_h V\rho/\mu$;
N_{St}, Stanton number, $h/\rho Vc$;
p, static pressure;
P^*, $p + (\rho_l g\, x \sin \varphi)/g_c$;
q'', heat flux;
T, temperature;
V, average liquid velocity;
x, flow length from start of heating;
μ, dynamic viscosity;
ρ, density;
τ, shear stress;
φ, angle of flow channel with horizontal.

Subscripts
b, bulk liquid;
bo, subcooled boiling;
l, saturated liquid;
n, running index;
nb, non-boiling;
o, condition at start of heating;
sat, saturation
w, heater wall;
1, 2, channel boundary indices.

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1. INTRODUCTION

SUBCOOLED nucleate boiling occurs when a liquid whose bulk temperature is below its saturation temperature is heated from a surface which is maintained at a temperature above the liquid saturation temperature. A thin layer of liquid next to the heating surface will then be superheated, and vapor bubbles may form in this layer. As these bubbles grow, they encounter subcooled liquid from the main stream and collapse, transferring their latent heat to the colder liquid. This growth and collapse process is repeated, with the result that there is no net flow of vapor from the surface, but there is a certain volume of vapor in the proximity of the heat-transfer surface at any time.

From the point of view of the designer, there are four major aspects to the subcooled boiling process for internal-flow applications: (1) the variation of heater surface temperature with heat flux, (2) the peak heat flux which can be accommodated in nucleate boiling, (3) the static pressure changes in the heated channel, and (4) the vapor volume per unit area of the heater. The first of these problems has been rather well treated in the literature, and adequate correlations are available. Near the peak heat flux, however, where a transition to film boiling occurs with a sudden, large increase in the surface temperature, conditions are not so well understood. Nevertheless, this latter feature has also received a great deal of attention lately, both in the form of empirical correlations and approximate analyses.

The present paper deals primarily with the static pressure variations which occur during steady-state, forced-convection, nucleate boiling of a subcooled liquid. It will be seen, however, that the accompanying variations in vapor volume must also be considered if the static pressure changes are to be treated adequately.

Inference may tentatively be made, from experience with single-phase flow, that there is an entrance effect in subcooled boiling flow which causes deviations of the various thermal and dynamic quantities near the place where boiling starts from more fully established conditions downstream. The present study is concerned chiefly with the condition of fully established subcooled boiling rather than with conditions in the entrance region.

Local values of the pressure drop during subcooled boiling have been reported by Sabersky and Mulligan [1]. Acting on a suggestion by H. S. Tsien that bubbles formed during subcooled boiling might be analogous to wall roughness in their effect on the thermal boundary layer, they reasoned that Reynolds' analogy between the transfer of heat and momentum, $f/2 = N_{St}$, should be applicable to subcooled boiling heat transfer.† They concluded from their results that this analogy is reasonably good over the limited range of Reynolds number (140 000–380 000) and Prandtl number (1·0–2·0) which they investigated. The analogy should be applicable either if the laminar sublayer is nonexistent (because of bubble activity, in this case) or for a fluid with a Prandtl number near unity, but the limited range of Prandtl number did not permit a distinction to be drawn between the two effects [3].

Static pressure measurements during subcooled boiling have also been reported by a number of other investigators, but none of these was able to find a general correlation which permitted extrapolation to other than their specific test conditions. The empirical pressure drop correlations proposed by Reynolds [4] and by Owens and Schrock [5, 6] will be compared with our results, as will the vapor-volume measurements of Costello [7, 8]. Kreith and Summerfield [9], Buchberg *et al.* [10] and Rohsenow and Clark [11] have reported the total pressure drop experienced along a channel in which subcooled boiling takes place, while Costello [7, 8] has reported local measurements. For reasons which will be discussed presently, none of these investigations is suitable for checking the present results except in the most general way.

2. ANALYSIS

Equations of motion will now be developed for flow in a closed channel which has a constant cross-sectional area and which is inclined at an angle φ to the horizontal. The channel may have one or more boundaries, at least one of which is a subcooled-boiling heat-transfer surface, while the remainder are adiabatic surfaces. The wetted periphery of each of these boundaries does not change in the axial direction of the channel, nor does the liquid mass flow rate or inlet temperature vary with time. In addition, the heat flux is constant with time and with position along the length of any of the boiling heat-transfer surfaces.

The following simplifying assumptions are made:

(1) The pressure of the system is considerably below the critical pressure; thus, the vapor density is very much less than the liquid density at any point in the channel.
(2) The time-and-space-averaged axial velocity of any vapor bubble is equal to or less

† Recent work [2], under the direction of the senior author of [1], has shown that Reynolds' analogy is not valid for rough tubes, even at a Prandtl number of unity. However, in the present paper, the validity of the analogy for forced-convection, subcooled boiling will be demonstrated by means of a different line of reasoning and additional experimental evidence.

than the time-and-space-averaged axial velocity of the liquid.

(3) The time-averaged fraction of the cross-sectional area occupied by the vapor is equal to or less than the time-averaged fraction occupied by the liquid.

(4) The densities of the liquid and vapor do not vary across the channel at any one axial position.

(5) Because of the growth and collapse of vapor bubbles during nucleate boiling, many of the variables which are important to the analysis vary with time. In particular, the area fractions, wall shear stress, and static pressure all experience high-frequency changes with time. It is assumed that time-averaged values of these quantities can be defined and, where the quantities are measured, that the measured values are these time averages.

(6) The liquid is in turbulent flow.

The equation of continuity under the above assumption is:

$$\rho_l V_l A_l = \rho_0 V_0 A_0, \tag{1}$$

where the definitions of the terms are given in the Nomenclature. The equation of axial momentum under the given assumptions and with the equation of continuity becomes:

$$-\left(\frac{\mathrm{d}p^*}{\mathrm{d}x}\right) = \frac{1}{A_0}(\tau_1 C_1 + \tau_2 C_2 + \ldots + \tau_n C_n) - \frac{V_l^2 A_l}{g_c A_0}\left(\frac{\mathrm{d}\rho_l}{\mathrm{d}T_b}\right)\left(\frac{\mathrm{d}T_b}{\mathrm{d}x}\right) - \frac{\rho_l V_l^2}{g_c A_0}\left(\frac{\mathrm{d}A_l}{\mathrm{d}x}\right). \tag{2}$$

Defining a friction factor for each boundary in the form

$$f_n = \frac{\tau_n}{\rho_l V_l^2/2g_c}, \tag{3}$$

equation (2) reduces to

$$-\frac{\mathrm{d}p^*/\mathrm{d}x}{\rho_l V_l^2/2g_c} = \frac{1}{A_0}(f_1 C_1 + f_2 C_2 + \ldots + f_n C_n) - \frac{2A_l}{A_0}\left(\frac{1}{\rho_l}\frac{\mathrm{d}\rho_l}{\mathrm{d}T_b}\right)\left(\frac{\mathrm{d}T_b}{\mathrm{d}x}\right) - \frac{2}{A_0}\left(\frac{\mathrm{d}A_l}{\mathrm{d}x}\right). \tag{4}$$

The first term on the right-hand side of this equation shows the frictional effect, while the remaining terms are acceleration components. The evaluation of the friction factor during local boiling will be discussed in detail in the next section. The second term in equation (4) is the same as for single-phase flow, except that there is an area ratio included. The third term represents the effect of the acceleration of the liquid which occurs when the vapor volume changes in the axial direction.

It is well known that the convective conductance for subcooled-boiling can be ten to fifty times greater than for single-phase heat transfer for the same values of liquid temperature, liquid velocity, and system pressure. The reasons for this large increase in convective conductance have been the subject of many investigations and analyses, some of which are based on the following models of the boiling process: (1) microconvection model of Gunther and Kreith [12] and of Ellion [13], (2) "vapor–liquid exchange action" model of Forster and Greif [14] and (3) sequential rate process model of Bankoff [15, 16].

An explicit or implicit assumption used in the development of each of the above models is that the boiling heat-transfer process is entirely turbulent, i.e. that heat transfer by conduction is negligible compared to heat transfer by turbulent mixing near the heat-transfer surface as well as in the main flow stream, and that the shear stress due to viscous forces is negligible compared to turbulent shear forces near the heat-transfer surface as well as in the main flow stream. This assumption is also sufficient for Reynolds' analogy [17] which states

$$f/2 = N_{St}. \tag{5}$$

Further discussion and comparison of these models may be found in [18].

The main objective of this investigation is to show the applicability of this analogy to forced-convection, subcooled boiling. Succeeding paragraphs will describe how the subcooled-boiling friction factor was found from equation (4) after calculation or measurement of the other quantities in that equation. The friction factor so determined will then be compared with the value predicted by the Reynolds' analogy.

3. EXPERIMENTAL METHOD

3.1 *Experimental apparatus*

Figure 1 shows a schematic drawing of the atmospheric pressure heat-transfer loop [18], in which distilled water flows through the following major components: (1) stainless-steel storage tank, (2) positive displacement 5 gal/min pump driven by a continuously variable-speed motor, (3) tube-in-shell, counter-flow cooler for rough control of the inlet water temperature, (4) flow meter with two floats of approximately 2 and 5 gal/min capacity, respectively, (5) two 4-kW electric immersion preheaters with continuous adjustment from 0 to 8 kW, (6) a 2-kW preheater controlled semi-automatically by a recording–controlling, self-balancing potentiometer, on which inlet water temperature is continuously recoredd and by which the preheater is controlled, and (7) the test section, which will be described below.

All of the above components and the connecting tubing have stainless-steel or Teflon wetted parts, with a few exceptions such as the copper bus-bars, in order to maintain the purity of the distilled water in the loop. Before each experimental run, the water was tested for purity by measuring its electrical resistivity, which was required to be in excess of 10^6 Ω cm. Deaeration was then accomplished by prolonged boiling in the storage tank, with the result that the usual gas content at the start of a run was about 1·3 ml/l. At the conclusion of the runs, the resistivity always exceeded 5×10^5 Ω cm and the gas content never exceeded 3 ml/l.

In the test section, water flows upward through an annular passage between the heating elements and an outer tube, as shown schematically in Fig. 2. The heating element consists of a $\frac{3}{16}$-in o.d., 0·005-in wall, stainless-steel tube, 11·3 in long, with copper tubes silver soldered to each end. Direct-current resistance heating in the stainless-steel tube produces a constant heat flux from the tube to the water in the annulus. The power dissipation from the test section is measured with a voltmeter, ammeter, and appropriate shunts. The over-all temperature increase of the water is used to obtain an energy balance. The outer tube is a $\frac{7}{16}$-in o.d., 0·012-in wall, stainless-steel tube, 24 in long, with pressure taps located as shown in the figure.

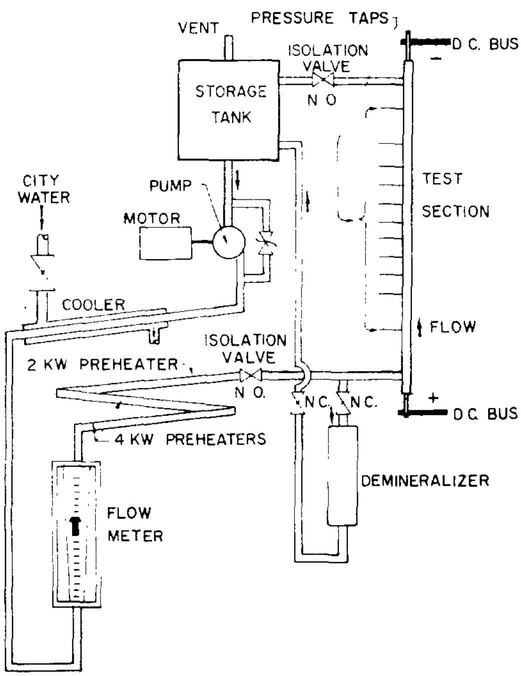

FIG. 1. Schematic drawing of heat-transfer apparatus.

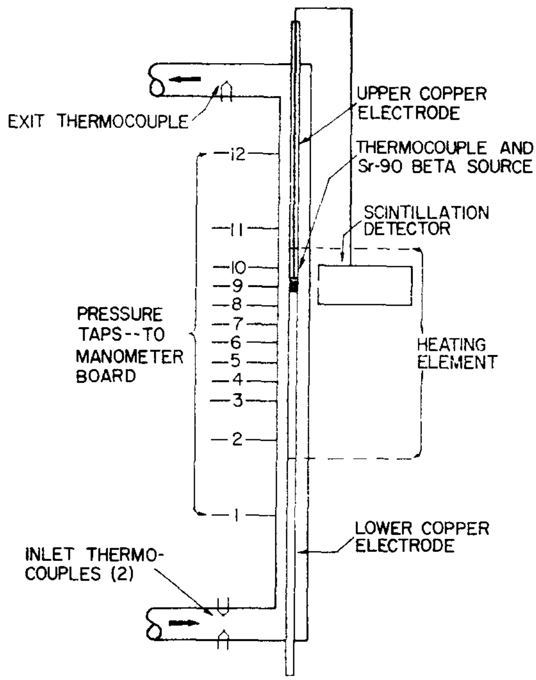

FIG. 2. Test-section instrumentation.

A 0·1 mc, Sr^{90} β-source is placed in the center of the inner tube opposite a scintillation detector. This detector is mounted in an apparatus allowing it to be moved axially between pressure taps 4 and 10, while the source is mechanically connected to the detector in such a way that the same part of the source is opposite the detector regardless of the position of the detector. The use of the radioactive source and the scintillation detector will be discussed below.

Three calibrated iron–constantan thermocouples are used with a precision, portable potentiometer; one to measure the water temperature as it enters the test section, one to measure the water temperature as it leaves the test section, and one located in the center of the heater tube, 1 in above the radioactive source, to measure the inside wall temperature of that tube. The bulk temperature of the water at any axial location within the test section is determined from the inlet and outlet thermocouple readings, while the outside wall temperature of the heat-transfer surface is determined from the inner wall thermocouple and the known heat flux. In addition to these three thermocouples, a fourth one is also located at the inlet to the test section and is connected to the recording-controlling potentiometer used to control the 2-kW preheater.

A twelve-tube manometer bank is used to measure the pressures at the pressure taps shown in Fig. 2. The static pressures at taps 2–10 are compared with the pressure of compressed air in a large tank. The indicating fluid used in these nine tubes is Meriam Fluid D-8325 (specific gravity of 1·75). The gage pressure of the compressed air in the tank, the over-all pressure drop in the test section (taps 1 and 2), and the gage pressure of the test section (tap 11) are also measured, using mercury as the indicating fluid.

The vapor volume per unit heater area is determined by β-attenuation in the liquid–vapor mixture, utilizing the Sr^{90} β-source, scintillation detector, preamplifier, linear amplifier with a pulse-height selector, precision ratemeter, and strip-chart recorder. Calibration was done at a previous time [19], the results of which serve to determine the vapor volume per unit heater area as a function of the ratio of the count rate during boiling to the count rate without boiling (i.e. zero vapor volume), and the bulk liquid temperature.

3.2 *Data reduction*

The friction factor for subcooled boiling can be calculated from equation (4):

$$\frac{f_1}{2} = \frac{g_c A_0}{\rho_l V_l^2 C_1}\left(-\frac{\mathrm{d}p^*}{\mathrm{d}x}\right) - \frac{C_2}{C_1}\frac{f_2}{2} - \frac{A_l}{C_1}\left(-\frac{1}{\rho_l}\frac{\mathrm{d}\rho_l}{\mathrm{d}T_b}\right)\left(\frac{\mathrm{d}T_b}{\mathrm{d}x}\right) - \left(\frac{\mathrm{d}a}{\mathrm{d}x}\right). \tag{4a}$$

The evaluation of most of the quantities in this equation is clear from their definitions, but a few should be explained at this point. Properties, including the saturation temperature, are found from the experimentally determined absolute pressure at given locations along the test section.

The static pressure gradient $(-\mathrm{d}p^*/\mathrm{d}x)$ is calculated from the measured pressures at taps 3–10. All of the values given in this paper are for a point 6·5 in from the start of heating, which is approximately the mid-point of the heater, and have been obtained by using a weighted first-order least-squares polynomial approximation [20]. The weighting function is arbitrarily chosen to be a parabola with a value of unity at the point of interest and a value of one-half at taps 4 and 9 (2·5 in on either side of the mid-point).

The outer-wall friction factor f_2 is assumed to have the isothermal value† corresponding to a Reynolds number based on the liquid bulk velocity, corrected for the actual liquid flow area A_l and on a hydraulic diameter, similarly corrected for the vapor-volume measurements. There are indications that the shear stress on the outer tube of an annulus without heating is only about 88 per cent of that on the inner tube [21], but this ratio is not necessarily applicable to the present situation. In any event, a 12 per cent change in this quantity leads to only a 4 per cent change in the boiling friction factor on the inside tube.

As has been explained earlier, the bulk-liquid temperature gradient is known to be linear because the heat flux along the tube is constant.

† Experimental data and a further discussion of isothermal friction factors may be found in [18].

Finally, the last term in equation (4a) is evaluated directly from the vapor-volume measurements.

4. DISCUSSION OF RESULTS

4.1 *Subcooled-boiling pressure drop*

The experimentally determined local boiling friction factors are plotted as a function of Stanton number and compared with the analytically deduced relation, $f/2 = N_{St}$, in Fig. 3. The ranges of the experimental variables are:

(1) Heat flux: 10^5 to 7×10^5 Btu/h ft^2
(2) Velocity: 2–10 ft/s
(3) Subcooling: 32–120°F
(4) Pressure: 15–21 lbf/in^2 a

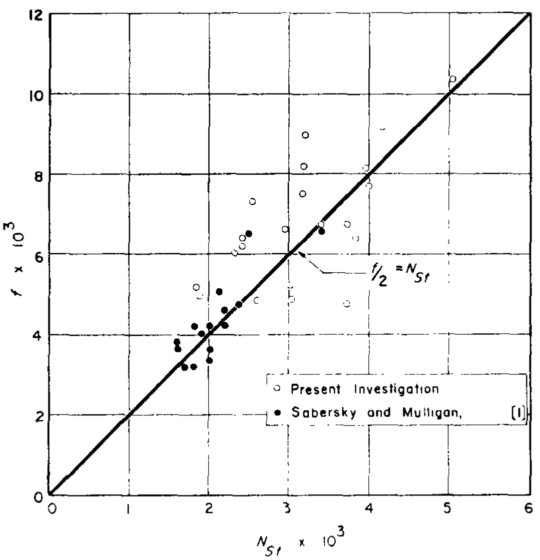

FIG. 3. Friction factor during subcooled boiling of water at 15–21 lbf/in^2 a, 10^5 to 7×10^5 Btu/h ft^2, 2–10 ft/s and 32–100°F subcooling.

An error analysis indicates that an uncertainty of ± 33 per cent in friction factor and ± 6 per cent in Stanton number may be expected, which may be compared with the indicated scatter of $+39$ per cent and -20 per cent in Fig. 3 from the analogy. The major contribution to the expected uncertainty is the subtraction of the outer-wall friction term from the total friction. This uncertainty could have been reduced by boiling on only one surface, as was done in the experiments of Sabersky and Mulligan [1] and of Owens and Schrock [5, 6], but the annular shape was chosen because of the relative ease of measuring vapor volume.

The data of Sabersky and Mulligan [1] are also shown in Fig. 3. These data are for three values of heat flux, from 5×10^5 to 10^6 Btu/h ft^2; three values of pressure, from 65 to 265 lbf/in^2 a; a range of water subcooling, from 59 to 82°F; and for a constant water velocity of 18·5 ft/s. Presumably the vapor-volume effect shown in equation (4) was accounted for by the measurement of liquid velocity with Pitot tubes at successive axial locations. Knowledge of the liquid velocity and the continuity equation makes a direct measurement of vapor volume unnecessary.

There are some doubts expressed by Sabersky [3] whether the results of their data indicated an applicability of Reynolds' analogy because the water in these tests was at temperatures which corresponded to Prandtl numbers relatively close to unity or because the bubble activity removed the effect of a laminar sublayer. In the present investigation, the Prandtl number, based on the bulk water temperature, varied from 2·0 to 4·5, which indicates that the latter reasoning is correct.

Local values of the apparent friction factor (see definition in the Nomenclature) and the Stanton number have been calculated from the data of Owens and Schrock [5, 6] and from the data of Reynolds [4] for four typical runs, each. These are shown in Fig. 4, from which it may be seen that, for low values of Stanton number (i.e. in the region of the test section where the subcooling is the greatest), these data approach the predicted relation as an asymptote. As the Stanton number is increased (i.e. subcooling decreased), the apparent friction factor increases faster with Stanton number than predicted.

It must be remembered, however, that these friction factors have been computed without corrections for the increasing vapor volume, because vapor-volume data were not obtained by Owens and Schrock or by Reynolds. Although no general method is available for predicting vapor volume during subcooled boiling, some of the runs of Owens and Schrock lie in the range of variables for which Griffith *et al.* [22] have measured and correlated this quantity. If their correlation is used for run 253 of Owens and Schrock's pressure-drop data, for example, the agreement with the analogy is quite good (Fig. 4).

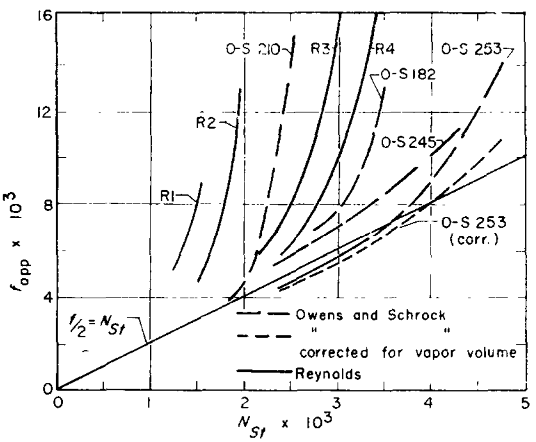

FIG. 4. Apparent friction factors calculated from data of Owens and Schrock [5, 6] and Reynolds [4] without correction for vapor volume (except for run O-S 253, as shown).

The system pressures for all of the tests reported by Reynolds were below the minimum pressure required for the prediction of the vapor volume by the method of Griffith *et al.* Therefore, his results could not be corrected for vapor volume. However, the similarity of his results to the uncorrected results of Owens and Schrock indicates that the deviation from Reynolds' analogy may also be caused by the vapor-volume effect on the pressure drop.

An investigation was made into the possibility that the data of Costello [7, 8] would further substantiate the applicability of Reynolds' analogy to the subcooled-boiling pressure-drop phenomenon. No conclusion can be made, however, for two reasons. The first is that Costello found the pressure drop varied appreciably with time for the majority of his tests, and, for these tests, he has reported only the maximum values of these fluctuating pressure drops. It is not felt that these maximum values of the pressure drop are representative for the purposes of testing the analogy. The second reason is that, for the tests which did not indicate fluctuating pressure drops, the scatter in the data is such that the pressure gradient cannot be obtained with sufficient accuracy.

Limited comparison between the results of the present investigation and those of Costello can be made, however. Although most of the test conditions are nearly the same, the instrumentation and the test procedure have been improved considerably, and the pressure-drop readings of the present investigation do not show any variation over the half-hour running time at a given set of experimental conditions.

The data of Kreith and Summerfield [9], of Buchberg *et al.* [10] and of Rohsenow and Clark [11] are for the over-all pressure drop across a test section in which subcooled boiling takes place. Local pressure gradients, which are necessary to calculate local friction factors, cannot be found from these data. Therefore, no comparison is made with their results.

4.2 *Vapor-volume measurements*

The variations of vapor volume with heat flux for four values of water subcooling are shown in Fig. 5. The calculated heat flux at which local boiling commences and the predicted peak heat flux [23] are shown for each curve. The variation of vapor volume with water velocity at constant heat flux and nearly constant water subcooling and system pressure is shown in Fig. 6. The expected uncertainty in the vapor-volume measurement is $\pm 0{\cdot}0016$ in^3/in^2.

As may be seen from these figures, the variable which influences the vapor volume the most is the liquid subcooling. Because curves such as these are not too useful for prediction purposes, many attempts were made to find a single parameter (involving the heat flux, water subcooling, water velocity and system pressure) which would correlate the vapor-volume data, but no such parameter could be developed.

The vapor-volume values reported by Costello [7, 8] seem to agree reasonably well with those found in the present investigation, although there is considerably more scatter in his results. A direct comparison of these data is made at a water bulk temperature of 140°F and a water velocity of 4 ft/s in Fig. 5.

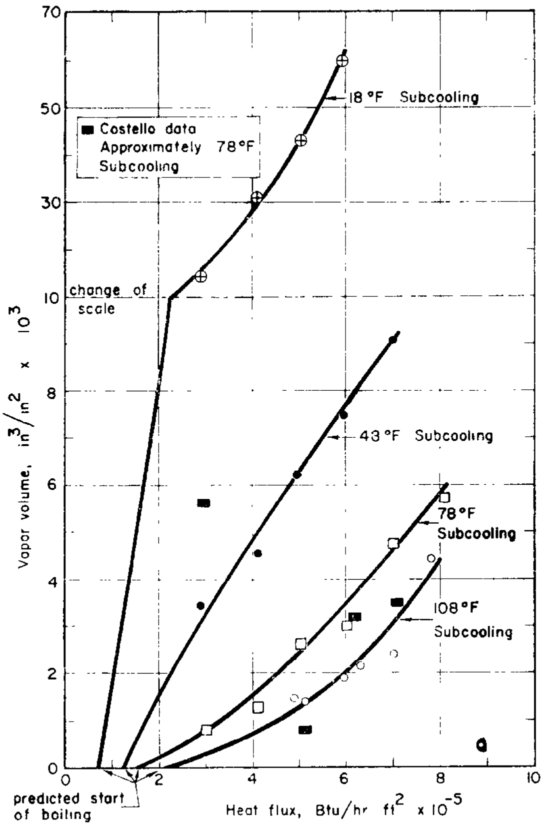

FIG. 5. Variations of vapor volume with heat flux and subcooling at 16·4 lbf/in^2 a, 4 ft/s. Predicted peak heat flux for 18°F subcooling is 7·7 $\times$ 10^5 Btu/h ft^2; for 48°F, 10·0 $\times$ 10^5 Btu/h ft^2; for 78°F, 12·3 $\times$ 10^5 Btu/h ft^2; and for 108°F, 14·6 $\times$ 10^5 Btu/h ft^2.

The correlation for vapor volume which is recommended by Griffith *et al.* [22] is

$$a = \frac{q''_{bo}\, N_{Pr,l}\, k_l}{1{\cdot}07\, h^2_{nb}\,(T_{sat} - T_b)}. \tag{6}$$

Table 1 gives a comparison between values calculated from the above equation and those measured at various values of heat flux and water subcooling. This table shows that the correlation predicts values of vapor volume which differ from the measured values by an order of magnitude for the high values of water subcooling. For lower values, the agreement is better, with a fortuitous exact agreement at the

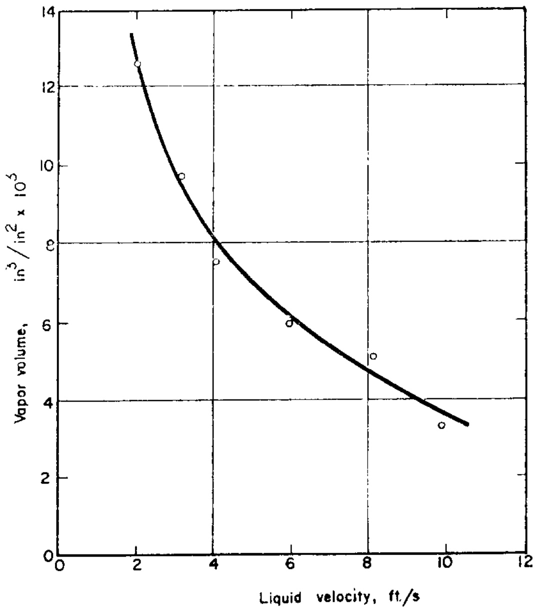

FIG. 6. Variation of vapor volume with velocity at 6·0 $\times$ 10^5 Btu/h ft^2, 15–21 lbf/in^2 a and 45–55°F subcooling.

last value of heat flux and water subcooling. It must be emphasized, however, that Griffith *et al.*, did not recommend their correlation for pressures below approximately 300 lbf/in^2 a, and this comparison is not made to cast doubt on the accuracy of their work or on the validity of their conclusions.

TABLE 1. *Comparison of vapor-volume correlation proposed by Griffith* et al. [22] *with values measured in present investigation*

Heat flux (Btu/h ft^2 $\times$ 10^{-5})	$(T_{sat} - T_b)$ (degF)	Bulk water velocity (ft/s)	Vapor volume: Measured (in^3/in^2 $\times$ 10^3)	Vapor volume: Calculated (in^3/in^2 $\times$ 10^3)
3·0	108	4·0	< 0·5	7·0
6·0	108	4·0	1·9	30
3·0	78	4·0	0·8	7·9
6·0	78	4·0	3·0	26
3·0	48	4·0	3·4	11
6·0	48	4·0	7·5	30
3·0	18	4·0	15	29
6·0	18	4·0	60	60

4.3 *Entrance effects*

In virtually all of the experimental tests during which the subcooled-boiling pressure drop was measured, an entrance effect is noticeable. Although the experimental apparatus does not permit the measurement of vapor volume within 4 in of the place where heating started, the variation of the vapor volume beyond this position, together with the wall-temperature and pressure drop measurements, strongly indicate that events near the start of heating are not at all the same as events further downstream. Fig. 7 shows typical variations of the heater wall temperature, vapor volume and pressure drop with the heater length. Also shown in this figure is the heater wall temperature which was measured at a later date under the same experimental conditions, but with the radioactive source not in position. The thermocouple could then traverse the entire length of the heating element.

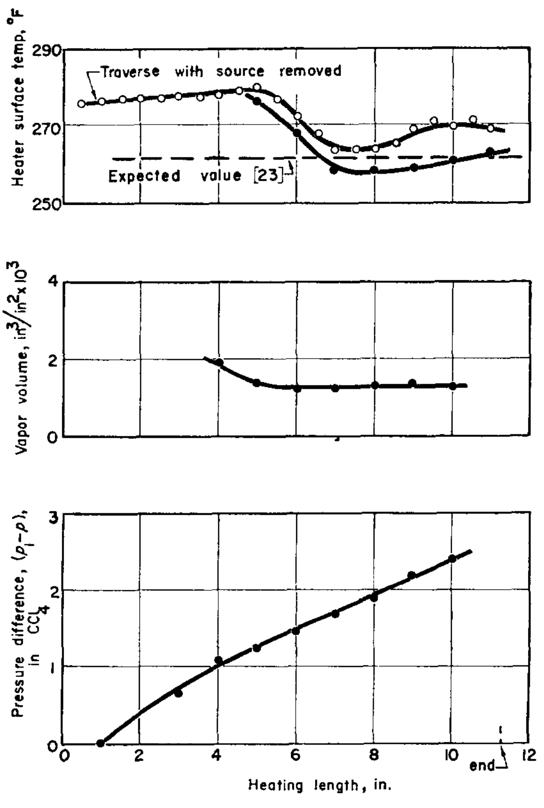

FIG. 7. Variation of heater surface temperature, vapor volume and static pressure with heating length at 5×10^5 Btu/h ft², 4 ft/s and bulk temperature from 86°F at inlet to 121°F at outlet.

Near the entrance, the wall temperature reaches a maximum value and then decreases to the value expected for subcooled-boiling heat transfer [23].† The vapor volume likewise increases to a maximum and then decreases to a fairly constant value; this effect may be caused by the higher values of liquid superheat near the heat-transfer surface. The pressure decreases relatively more in the entrance region than further downstream, probably reflecting the larger value of the vapor volume in the entrance region. It also appears that there is no pressure recovery in the deceleration region when the vapor volume is decreasing.

These observed entrance effects might be explained either as being a result of an anomaly of the test section or as being caused by the development of a steady temperature profile through the bulk of the liquid. Certainly, at the very beginning of the heated length, a non-boiling thermal boundary layer must develop until there is a region of superheated liquid in which bubbles may grow. There may then develop a region where there is competition between single-phase heat transfer, which would tend to continue the thermal boundary layer growth, and boiling heat transfer, which would tend to retard the thermal layer growth by bubble activity. It would be expected that the wall temperature for this region would lie between that expected for single-phase heat transfer and that for fully established subcooled boiling. However, it is difficult to understand how this region could extend downstream as far as indicated by the wall temperature profiles (Fig. 7) and then suddenly end, with subcooled boiling being fully established thereafter. Clearly, the friction factors and vapor volumes reported in this paper are applicable only to the fully established subcooled-boiling conditions; the entrance effects associated with subcooled boiling warrant further investigation.

† A similar wall temperature profile at the beginning of a test section has been reported by Schrock and Grossman [24]; however, their tests were made with bulk boiling rather than subcooled boiling.

5. CONCLUSIONS

The friction factor for subcooled-boiling heat transfer agrees reasonably well with the Reynolds' analogy prediction $f/2 = N_{St}$. This analogy depends on the assumption that, during subcooled boiling, the high degree of liquid agitation due to the activity of the bubbles virtually destroys the laminar sublayer, which is the substantial contributor to the thermal resistance during single-phase heat transfer. This assumption also leads to Reynolds' analogy between energy and momentum transport.

The measured friction factor and the predicted variation of friction factor with Stanton number were also found to agree with the data of Sabersky and Mulligan. Deviations from the analogy were found for the data of Owens and Schrock and of Reynolds when their apparent friction factors were compared with the Stanton numbers. However, calculations which estimate the influence of the vapor volume on the friction factor indicate that the deviations can probably be attributed to the effect of vapor volume on the pressure drop.

The variation of the vapor volume per unit area with heat flux, subcooling and velocity are presented. Fair agreement of these data with that of Costello was found.

An entrance effect can be seen in virtually all of the experimental runs. Near the entrance the heater wall temperature is higher, the vapor volume greater and the pressure gradient larger than for the fully established local boiling conditions downstream.

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Résumé—Pour déterminer la perte de charge, les auteurs utilise l'analogie de Reynolds entre les transferts de chaleur et de quantité de mouvement. Les mesures de pression statique, du volume de vapeur par unité de surface et de la température de paroi chauffée, permettent, dans le cas d'une disposition annulaire, de calculer le coefficient de frottement local et le nombre de Stanton local. A partir de ces valeurs, on montre que l'analogie donne une bonne approximation du coefficient de frottement si l'on fait une correction pour tenir compte des effets d'accélération. Toutefois cet accord n'est pas vérifié à l'entrée de la partie chauffée; des divergences sont relevées entre les quantités mesurées à l'entrée et plus en aval.

Zusammenfassung—Die Reynoldsanalogie zwischen Wärme- und Impulsaustausch wird zur Berechnung des Druckabfalls auf die Zwangskonvektion beim Verdampfen in unterkühlter Flüssigkeit angewandt. Aus Messungen des statischen Druckes, des Dampvolumens pro Flächeneinheit und der Heizwandtemperatur, wie sie an einem Ringraum durchgeführt wurden, lässt sich der örtliche Widerstandsbeiwert und die örtliche Stantonzahl errechnen. Die Analogie ergibt eine gute Annäherung für den Widerstandsbeiwert, wenn für die Beschleunigungseinflüsse eine Korrektur vorgenommen wird. Für den Beginn der Heizstrecke gilt diese Übereinstimmung jedoch nicht; örtliche Abweichungen der Messwerte von jenen stromabwärts finden sich im Einlaufgebiet.

Аннотация—Аналогия Рейнольдса между переносом тепла и количества движения применена для вычисления перепада давления в случае вынужденной конвекции в переохлажденной кипящей жидкости. Экспериментальные измерения статического давления, объёма пара на единицу площади и температуры стенкм нагревателя в кольцевом пространстве позволяют вычислить локальные значения коэффициента трения и критерия Стантона. Из этих данных можно заключить, что аналогия даёт хорошее совпадение для коэффициента трения, если сделана поправка на эффекты ускорения. Однако, вблизи начала нагреваемого участка, во входной области, обнаружены отклонения измеренных местных величин от аналогии Рейнольдса.